The Effect of Second Order Refraction on Optical Bubble Sizing in Multiphase Flows

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In multiphase flow research and many industrial applications it is important to determine the bubble size and velocity. To achieve this, one of approaches is to utilize laser phase-Doppler anemometry. However, it was found that the second order refraction has great impact on PDA sizing method when the relative refractive index of media is less than one. In this paper, the problem of second order refraction is investigated and a model of phase-size correlation to eliminate the measurement errors is introduced for bubble sizing. As a result, the model relaxes the assumption of single scattering mechanism in conventional phase-Doppler anemometry. The results of simulations based on this new model by using Generalized Lorenz Mie Theory (GLMT) are compared with those based on the conventional method. An optimization method for accurately sizing air-bubble in water has been suggested.

Key Words: Phase-Doppler Anemometry, Scattering, Bubbly Flows, Particle Sizing

1. Introduction

Phase-Doppler Anemometry (PDA) is nowadays widely used for velocity measurement and particle sizing in experimental studies of multiphase flows. Various developments were made to extend the application area of PDA. The accurate measurement of droplet size is the utmost important since the bubble volume depends on the third power of the bubble diameter. To achieve high spatial resolution, it is often required to decrease the measurement volume size. Unfortunately, a simple reduction in the measurement volume size can cause the phenomena in conflict with the assumption of a uniform illumination. Consequently, the result suffers from error in sizing large particles due to the nonlinearity in the phase/diameter relationship because the beam intensity is practically nonuniform (Aizu et al., 1993; Saffman, 1986; Durst et al., 1994; Sankar et al., 1992; Grehan et al., 1991; Qiu and Sommerfeld, 1992; Qiu and Hsu, 1996; 1999; Tropea et al., 1994). Various methods to minimize the trajectory ambiguity have been proposed for classical geometry (Qiu and Sommerfeld, 1992; Qiu and Hsu, 1996; Tropea et al., 1994). However, all of above methods for the elimination of the measurement volume effect are based on the signal validation scheme, which may cause the complexity in the determination of measurement volume size. The consequence is that the calibration could be very complicated.

The recent development (Qiu and Hsu, 1999) introduced a new approach by taking both the refraction and reflection into account. It demonstrated successfully in measuring particles with refractive index greater than one. The Gaussian beam defect and slit effect can be eliminated according to the validation

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experiments. However, the previous simulations were only conducted for the case of refractive index greater than one. It was found that the second order internal refraction could have great impact on the particle sizing if the relative refractive index is less than one (e. g. in bubbly flows). In this study a method based on the recently developed dual mechanisms' scattering model (Qiu and Hsu, 1999) was further developed. In this method two scattering phases measured by photodetectors are used to calculate the particle size. It is found that although the spatial frequencies of first order refraction and surface reflection become identical at the optimized scattering angle as suggested by Qiu and Hsu (1999), the second order refraction can not be neglected, which has a different spatial frequency and motion direction. By utilizing the conversion factor of the second order refraction, the particle diameter can be solved numerically. To demonstrate the capability of the newly developed method, the model was simulated numerically by using Generalized Lorenz Mie Theory (GLMT). The optical parameters such as the measurement volume size, the focus lengths of the sending and receiving lenses, the size and shape of the receiving aperture, the particle size and its trajectory, and the phase conversion factors are used to analyze the performance of the new developed method.

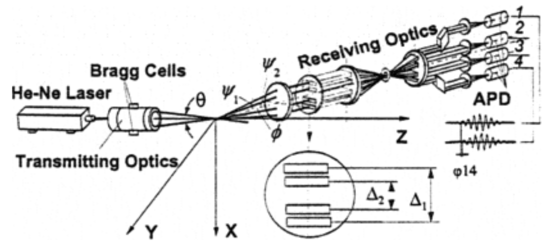

Fig. 1 Optical layout of four-detector phase Doppler anemometry: APD, avalanche photodiode

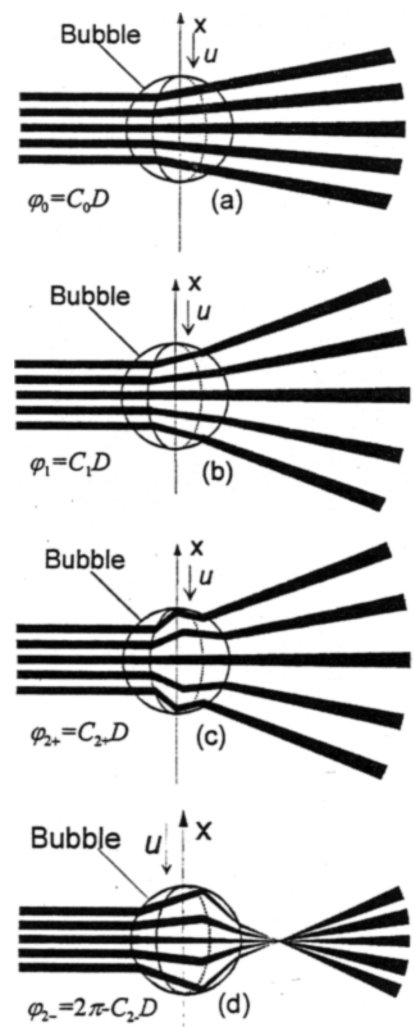

Fig. 2 Bubble scattering fringe patterns

2. Analytical Description

The schematics of the newly proposed phase-Doppler system is shown in Fig. 1, where the receiving optics consists of four detectors that are symmetrically orientated with elevation angles ψ_1 and ψ_2 for outer and inner detector pairs, respectively. The light scattering patterns from a bubble in the measurement volume of the PDA system are shown in Fig. 2 (a) - (d) for surface reflection, positive-side first order refraction, positive-side second order refraction and negative-side second order refraction, respectively.

The phase difference $\varphi 14$ and $\varphi 23$ between the two outer and inner detectors can be determined from the two Doppler signal pairs (see Fig. 1). The light scattering from a bubble in the measurement volume to a specific angle of receiving unit of a PDA system is shown in Fig.

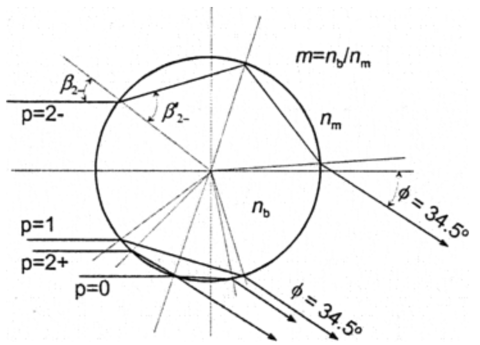

Fig. 3 Description of diversified incident angles and scattering patterns at y>0 side from different scattering mechanisms when the scattering angle is selected at 34.5° for $m=1/1.33$

3, where p equates 0, 1, 2+ and 2− denoting the surface reflection, first order refraction, and second order refractions in both positive and negative sides respectively.

It is clear that the measured phase is contributed by both reflection and refractions, which coexist in most cases. Therefore, the problem is not one of being dominated by either reflection or refractions but a combination of all of them. By using geometric approach, the phase-size conversion factors of different scattering mechanisms can be described as

$$C_p=\frac{720\tan(\psi)\cos(\tau_p)\sin\left(\frac{\theta}{2}\right)}{\lambda\sin(\phi)} \qquad (1)$$

where τ_p denotes the incident angle for each scattering mechanism ($\tau_p=\pi/2-\beta_p$) and θ, ϕ and ψ are as defined in Fig. 1.

By using previous research results, the surface reflection ($p=0$) and first order refraction ($p=1$) can be easily obtained as below

$$\tau_0=\frac{\phi}{2} \qquad (2)$$

$$\tau_1=\arccos\left(\frac{m^2\sin^2\left(\frac{\phi}{2}\right)}{1+m^2-2m\cos\left(\frac{\phi}{2}\right)}\right) \qquad (3)$$

where m and ϕ are relative refractive index and off-axis angle, respectively.

The relationship between the incident angle and optical parameters can be formulated by using geometric optics approach and solving a forth order polynomial equation. As a result the incident angles of second order refraction for both positive and negative sides are

$$\tau_{2+}=2\arccos(x_+) \qquad (4)$$

$$\tau_{2-}=2\arccos(x_-) \qquad (5)$$

where x_+, x_-, a, b, c, and d are described in Eqs. (6)-(10).

$$x_+=\frac{1}{2}\left(-\frac{(a-\sqrt{8y_1+a^2-4b})}{2}+\sqrt{\frac{(a-\sqrt{8y_1+a^2-4b})^2}{4}-4\left(y_1-\frac{(ay_1-c)}{\sqrt{8y_1+a^2-4b}}\right)}\right) \qquad (6)$$

$$x_-=\frac{1}{2}\left(-\frac{(a-\sqrt{8y_1+a^2-4b})}{2}+\sqrt{\frac{(a-\sqrt{8y_1+a^2-4b})^2}{4}-4\left(y_1-\frac{(ay_1-c)}{\sqrt{8y_1+a^2-4b}}\right)}\right) \qquad (7)$$

$$a=-m\cos\left(\frac{\phi}{4}\right),\ b=\frac{m^2-4}{4},$$

$$c=\frac{1}{2}m\cos\left(\frac{\phi}{4}\right),\ d=\frac{1-m^2\sin^2\left(\frac{\phi}{4}\right)}{4}; \qquad (8)$$

$$p=\frac{3(a\cdot c-4d)-b^2}{12},$$

$$q=\frac{-2b^3+9b\cdot(a\cdot c-4d)+27(d(4b-a^2)-c^2)}{216} \qquad (9)$$

$$\Delta=\frac{q^2}{4}+\frac{p^3}{27},$$

$$y_1=\left(-\frac{q}{2}+\sqrt{\Delta}\right)^{\frac{1}{3}}+\left(-\frac{q}{2}-\sqrt{\Delta}\right)^{\frac{1}{3}}+\frac{b}{6} \qquad (10)$$

The calculated phase-size conversion factors C_0, C_1, C_{2+} and C_{2-} are shown in Fig. 5. As indicated by previous researches, for a particle passing through the focused beams along different trajectories, the balance between the reflected and refracted rays will vary. This trajectory dependence can lead to significant errors. In Fig. 5, it is found that $C_0=C_1$ at $\phi=82.5$ which is called the optimized angle, as described by Qiu and Hsu (1999). It is important to know that C_{2+}, C_0, and C_1 are also identical at this optimized angle for m=1/1.33. From Fig. 4 we also know that the fringe patterns of reflection, first and second order refractions in positive side move in the same direction. Therefore, the contribution of the three scattering rays becomes identical and we can use an overall intensity and phase-size conversion factor C_0 to represent it. Since the nega-

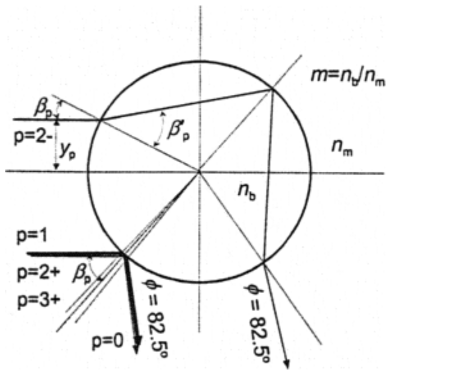

Fig. 4 Description of identical incident angles (β_p) at y >0 side from different scattering mechanisms when the optimized scattering angle (ϕ=82.5°) is chosen for m=1/1.33

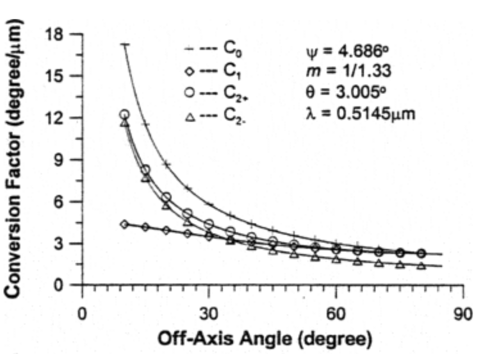

Fig. 5 Phase-Size Conversion Factors for Different Scattering Mechanisms

tive side second order refraction moves towards the opposite direction of the particle motion, a combined scattering model can be obtained by providing that the phase-size relations for a four detector system (Qiu and Hsu, 1999).

The relationship between the measured phases from multi-detectors and optical parameters can be described in Eq. (11), where $C1_p$ and $C2_p$ are the phase-size conversion factors for phases $\varphi 14$ and $\varphi 23$, respectively.

$$\frac{\left(-\cos\left(\frac{C1_{2-}D}{2}\right)\tan\left(\frac{\varphi 14}{2}\right)-\sin\left(\frac{C1_{2-}D}{2}\right)\right)}{\left(\tan\left(\frac{\varphi 14}{2}\right)\left(\cos\left(\frac{C1_0 D}{2}\right)-\cos\left(\frac{C1_{2-}D}{2}\right)\right)-\sin\left(\frac{C1_0 D}{2}\right)-\sin\left(\frac{C1_{2-}D}{2}\right)\right)} - \frac{\left(-\cos\left(\frac{C2_{2-}D}{2}\right)\tan\left(\frac{\varphi 23}{2}\right)-\sin\left(\frac{C2_{2-}D}{2}\right)\right)}{\left(\tan\left(\frac{\varphi 23}{2}\right)\left(\cos\left(\frac{C2_0 D}{2}\right)-\cos\left(\frac{C2_{2-}D}{2}\right)\right)-\sin\left(\frac{C2_0 D}{2}\right)-\sin\left(\frac{C2_{2-}D}{2}\right)\right)} = 0 \quad (11)$$

Table 1 Optical parameters

Parameters	Value	Unit
Wavelength (λ)	0.5145	μm
Radius of Beam Waist (r_0)	40	μm
Transmitting Angle θ	3.06	degree
Receiving Elevation Angle ψ_1	4.686	degree
Receiving Elevation Angle ψ_2	2.343	degree
Off-Axis Angle φ	82.5	degree
Relative Refractive Index m	0.7518	

Note that the conversion factor C_{2-} does not equal to the C_0 at the optimized off-axis angle. By solving Eq. (11) the bubble diameter can be determined.

3. Results and Discussion

To evaluate the performance of the new method, Generalized Lorenz Mie Theory (GLMT) was used to simulate MVE in comparison with that based on the conventional single scattering mechanism assumption. Simulations of the MVE were carried out for the system geometry as shown in Fig. 1 with the parameters given in Table 1. The equation used in (Qiu and Hsu, 1999) was adopted to determine the optimised off-axis angle. For the conventional method, only the inner detector pair is used in the simulation. The major direction of particle motion was in the Y direction, which is most critical in practical cases.

The results of simulations are shown in Fig. 6 to Fig. 10. As shown in Fig. 6, the measured diameters for both the conventional, first order approach and the newly developed methods are almost identical with an ideal diameter 40μm. This is because for a given droplet diameter smaller than the measurement-volume diameter, the second order MVE can normally be neglected. If the droplet diameter is larger than the diameter of the measurement volume, the MVE becomes significant as shown in Fig. 7 to Fig. 10. As shown in Fig. 7, a 60 μm air bubble passing through the measurement volume parallel to the Y axis at z=0 μm will yield a large-phase transition (>100% in this special case) owing to the

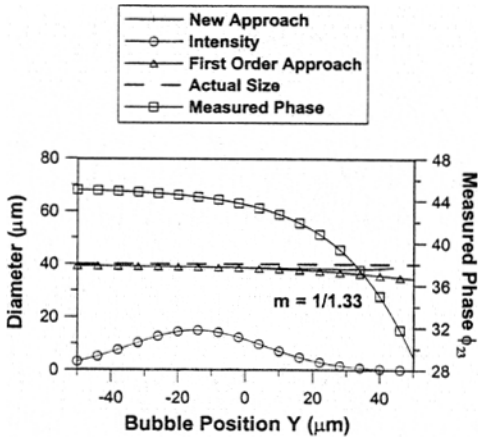

Fig. 6 Phase-measurement results among the conventional, the first order and the new approaches for a 40μm bubble in water moving along Y-direction

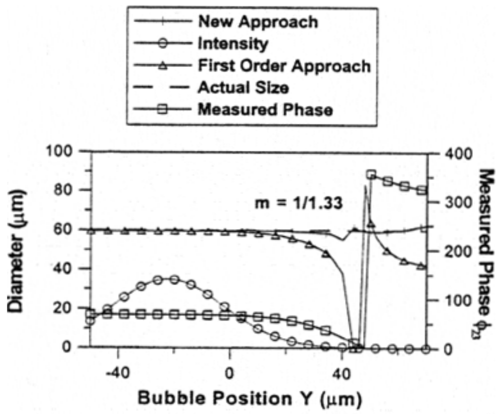

Fig. 7 Phase-measurement results among the conventional, the first order and the new approaches for a 60μm bubble in water moving along Y-direction

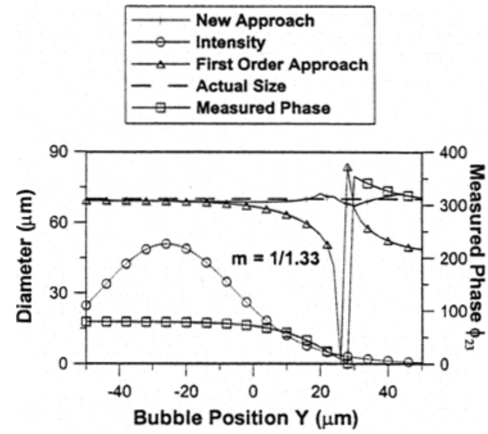

Fig. 8 Phase-measurement results among the conventional, the first order and the new approaches for a 70μm bubble in water moving along Y-direction

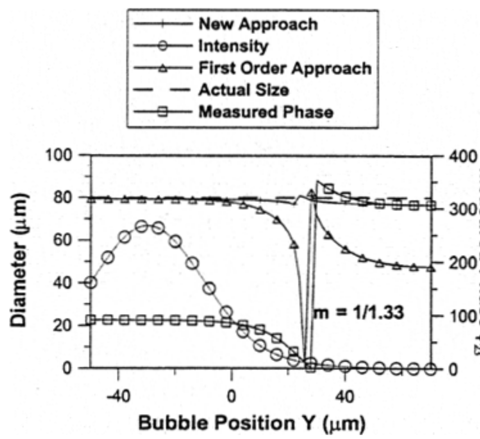

Fig. 9 Phase-measurement results among the conventional, the first order and the new approaches for a 80μm bubble in water moving along Y-direction

change of the scattering mechanisms from reflection to refraction. If the phase is determined by the conventional and first order approach methods, a large measurement error may occur. Only a very small fluctuation ($<3\%$) in the phase determination from using Eq. (11), hence the MVE, can be neglected. Fig. 8 shows slightly large fluctuations in the size-trajectory dependence for the new method. However, the fluctuations are still with 4% which is acceptable for most multiphase flow measurements. Figures 9 and 10 show similar results for 80- and 100 (m bubbles, respectively. Although the scattering alternating between reflection and refraction yields a large-phase change in the conventional method, an improvement can be observed if we take both first order refraction and reflection into account. However, more than 20% error can be found when $Y>20$ μm. Only very small fluctuation ($<4\%$) for the phase determination by using the new method can be observed.

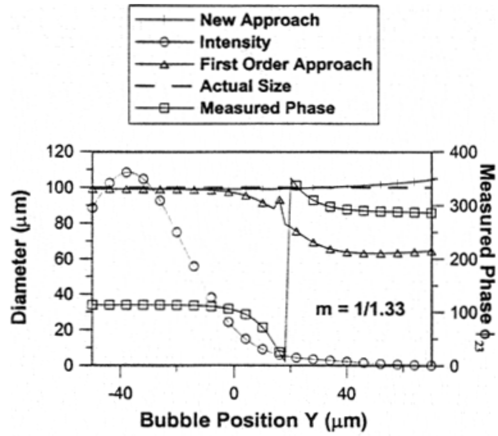

Fig. 10 Phase-measurement results among the conventional, the first order and the new approaches for a 80μm bubble in water moving along Y-direction

4. Conclusion

A further improvement of PDA measurement free from measurement volume effect was developed for bubble measurements. This new method can effectively eliminate the measurement volume effect including Gaussian beam defect and slit effect by using a four-detectors' PDA system which is very easy to be implemented in two-phase flow measurements. Because no strong validation scheme is necessary other than the signal-to-noise ratio, the measurement volume can be accurately determined by conventional methods, and hence, we have high accuracy in measuring the bubble volume flux and concentration. The performance of this new method was simulated by using GLMT. The newly developed model is especially suitable for sizing particles with refractive index less than one.

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